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1993 J. Phys. A: Math. Gen. 26 L575

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J. Phys. A: Math. Gen. 26 (1993) L575–L577. Printed in the UK

LETTER TO THE EDITOR

Cosmic strings from the string theory

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Received 2 March 1993

Abstract. The possibility of obtaining the cylindrically symmetric solution in the case of low energy string theory is investigated.

The low energy limit of string theory gives a set of equations similar to Einstein's equations for metric tensor and other fundamental fields. Recently it has been shown that this theory admits solutions with the structure of an extended object surrounded by an event horizon, the so-called 'black-string' [1]. The 'black-string' solution was also used to find new solutions representing waves travelling on the 'black-string' background [2].

The four-dimensional low-energy action obtained from string theory has the form [3, 4]:

$$S=\int d^4x\sqrt{-g}\left[-\frac{R}{\kappa^2}-2\frac{(\nabla\phi)^2}{\kappa^2}-\tfrac{1}{2}\,e^{-2\phi}F^2\right] \tag{1}$$

where $\kappa^2=8\pi G$, $F_{\mu\nu}$ is the Maxwell field associated with a U(1) subgroup of $E_8\times E_8$ or spin$(32)/Z_2$. We set the remaining gauge fields to zero. ϕ is the dilation field. This case corresponds to the bosonic sector of the field-theory limit of the superstring model. Extremizing action with respect to the U(1) potential A_μ, ϕ and $g_{\mu\nu}$ one gets the following field equations:

$$\nabla_\mu(e^{-2\phi}\,F^{\mu\nu})=0 \tag{2}$$

$$\frac{\nabla^2\phi}{\kappa^2}+\tfrac{1}{4}\,e^{-2\phi}F^2=0 \tag{3}$$

$$G_{\mu\nu}=-8\pi T_{\mu\nu} \tag{4}$$

where the energy-momentum tensor

$$T^{\mu\nu}=\frac{2}{\sqrt{-g}}\,\delta S/\delta g_{\mu\nu}$$

is given by:

$$T_{\mu\nu}=e^{-2\phi}\,F_{\mu\gamma}F^{\gamma}_{\nu}+g_{\mu\nu}\,e^{-2\phi}\,F^2-2\nabla_\mu\phi\nabla_\nu\phi+g_{\mu\nu}(\nabla\phi)^2. \tag{5}$$

The main subject of this letter is the investigation of the possibility of obtaining the cylindrically symmetric solution of the Einstein–Maxwell field coupled to the dilation field and finding the main features of it. The most general static cylindrically symmetric has the following form:

$$ds^2 = e^{2(\gamma-\psi)}(dt^2 - dr^2) - e^{2\psi}\, dz^2 - \alpha^2\, e^{-2\psi}\, d\phi^2 \tag{6}$$

where α, γ, ψ are only r-dependent. The Einstein field equations for this case are [5]:

$$\alpha'' = -8\pi G(T_0^0 + T_r^r) \tag{7a}$$

$$(\alpha\psi')' = -4\pi G(T_0^0 + T_r^r + T_\phi^\phi - T_z^z) \tag{7b}$$

$$(\alpha\gamma')' = -8\pi G(T_r^r + 2T_\phi^\phi) \tag{7c}$$

$$-\gamma''\alpha = \alpha\psi'^2 + 8\pi G T_\phi^\phi. \tag{7d}$$

A prime denotes a derivative with respect to r. In our case we assume that ϕ and A are only r-dependent and A has the z-directed component. If we assume that outside a radius r_0 components of $T_{\mu\nu}$ are negligible then one obtains vacuum solutions of the form

$$\alpha(r) = k(r - r_0) + \alpha(r_0) \tag{8a}$$

$$\gamma(r) = a \ln \alpha(r)/\alpha(r_0) + \gamma(r_0) \tag{8b}$$

$$\psi(r) = -b \ln \alpha(r)/\alpha(r_0) + \psi(r_0). \tag{8c}$$

It is also convenient to define the quantities [6, 7]:

$$T_1(r) = \int_0^r (T_r^r + T_\phi^\phi)\sqrt{-g}\, dr \tag{9a}$$

$$T_2(r) = \int_0^r (T_t^t - T_z^z)\sqrt{-g}\, dr \tag{9b}$$

$$T_3(r) = \int_0^r (T_r^r - T_t^t)\sqrt{-g}\, dr. \tag{9c}$$

Denoting the $r \to \infty$ limit of $2\pi r T_i(r)$ by τ_i we may express these parameters as:

$$\alpha(r_0) = \int_0^{r_0} (1 - 8\pi G T_3(r))\, dr$$

$$k = 1 - 4\pi G \tau_3(r)$$

$$\psi(r_0) = -4\pi G \int_0^{r_0} \frac{[T_1(r) + T_2(r)]}{\alpha(r_0)}\, dr$$

$$b = 2G\frac{(\tau_1(r) + \tau_2(r))}{k}$$

$$\gamma(r_0) = -8\pi G \int_0^{r_0} \frac{T_1(r)}{\alpha(r)}\, dr$$

$$a = -4G\frac{\tau_1(r)}{k}$$

where we used the boundary conditions $\alpha'(0) = 1$, $\gamma' = \psi' = 0$, $\alpha(0) = \gamma(0) = \psi(0) = 0$.

In general, relativity studies of line sources are motivated by their interesting features. Cosmic strings act as gravitational lenses. They double images of quasars and may be seeds for accumulating matter in galaxy formations. It is interesting to analyse the solution, treated as the exterior spacetime of an analogue of the cosmic string, and it will not be amiss to ask about the familiar features in the above case. It could be easily shown that the 'local deficit angle' is of the form

$$\delta(r)=2\pi\left(1-\frac{1}{\sqrt{-g_{rr}}}\frac{\partial}{\partial r}\sqrt{-g_{\phi\phi}}\right)=2\pi[1-k\,\mathrm{e}^{-\gamma}(1+b)]. \tag{10}$$

If we use the energy conservation law and set b equal to zero, one can see, in calculations to the first order, that $\delta(r)$ has no dependence on the impact parameter. Its value is $8\pi G\mu$. This is the same result as for gauge strings in general relativity [8].

Another important question concerning this problem is the enquiry about the stability of the solution. In order to have a closer look at it, we recall the brilliant conception of K Thorne, the C-energy. Integration equations (7) with respect to the first order in the dimensionless quantity $G\times$(mass per unit length) and substituting the solutions of the equations for the C-energy per unit of standard length inside the cylinder and on the standard hypersurface t – constant, we get

$$E(r_0)=\int_0^{r_0}2\pi\tilde{r}T_0^0\,\mathrm{d}r=\int_0^{r_0}\mathrm{e}^{-2\gamma}[2\,\mathrm{e}^{-2\phi}(A_z')^2-\mathrm{e}^{2\psi}(\phi')^2]2\pi\tilde{r}\,\mathrm{d}\tilde{r}. \tag{11}$$

When fields appearing in the above equation fulfill the condition that the C-energy is non-negative, one does not hesitate to identify it with the classical energy [5]. Because the field configuration under consideration satisfies the classical variational equations of motion, it is located at a local minimum energy condition. So we draw the conclusion that the static string configuration is situated at a local C-energy minimum and is stable to small perturbations.

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